When payoffs are given, choice of an action is determined by choice of a prob. dist. (To say you know A's probs and you know his vN payoffs — A may be you — is to say you know what he wants to choose. To say you know what he wants to choose is to say you know his (max) expectation; from which payoffs and/or probs can be inferred (Why bother?) if you know other things).

So the choice of a prob dist by a committee — or by a person who finds several prob dists reasonable and has no definite reason to regard one as more likely than another or as equally likely, or who is not confident of his reasons — is a decision problem, each ξ (dist) having payoffs corresponding to the expected value of the "best" action under that ~~action~~ dist for each of the other "reasonable" dists.

Threshold problems: several of the ξ's in question may give the same ordering, or the same optimum, or may order certain actions the same way but not others; disagreement over actions is constrained by disagreement over dists, but not in one-one fashion; it is not necessary to reach total agreement on dist. to agree at action

i.e. many rows in matrix below are identical and all but one can be ignored:

$\xi_1 \quad \xi_2 \quad \xi_3 \quad \ldots$

$\xi_1 \quad E\bar{x}(\xi_1)_{\xi_1} \quad \ldots$

ξ_2

$\xi_3 \quad E\bar{x}(\xi_3)_{\xi_1}$

$\bar{x}(\xi_1)$ is optimum x in original decision problem w.r.t. ξ_1.

Imagine choice committee made up into two subcommittees: Payoffs (Policy) and Probs (Intelligence).

1) Bayes' approach implies these committees reach conclusions separately, on separate data, turn over results to Decision-maker (Pres, NSC, Ops); or, Probs turn over results to Policy, which then produces unique choice (results in form of a single prob. dist.).

2) Minimax regret or minimax implies that Policy committee arrives at result ignoring Probs conclusions, just because they didn't agree totally.

3) How should Probs committee report its findings and disagreement? Why should own

Why "should" Probs committee report its degree of uncertainty
or range of possibilities, except in prob. terms?" (as RAND says).

(Ans: because D may want to give different weights to different "possibilities" — prob dists — depending on payoffs, the wants of Policy Committee. So Intell. should make 2 sorts of findings: a) Ambiguity, overall adequacy of info b) Relative likelihoods, of events and of prob. dists.

If (a) shows high ambiguity, D will want to make judgment of ρ and α, instead of just turning over (b) to Policy Committee.

(Or, D is his own Payoff Committee, with Outcome + Action Committee finding out possibly objective consequences of actions + events, Intell helping on predicting these, in addition to predicting ~~unknown~~ probs of events).


Military organization

The efficiency of a military system must always be judged in the light of a number of degradation-contingencies.

A sergeant is judged not only on how well his platoon performs but on <u>how</u> he runs it; specifically on (a) how well it performs under his direction <u>and</u> (b) how well it would perform <u>without</u> him (if he were killed, out of comm, replaced).

Aside from the problem of training subordinates for leadership positions (which <u>could</u> be done separately) platoon must be trained + run to operate well without comm and without normal leadership.

Still, with platoon this mainly amounts to ability to carry out a given decision, task; ability to be re-directed is less important.

30. The difference between maximin and minimax--between the values of the minorant and majorant games--is the least amount a player could gain by "finding out" the enemy--estimating his intentions correctly. In a normalized game, where the enemy doesn't know he will be found out, the actual gain might be much more: from maximin to maximax.

But even the minimum gain might be significant, if it is greater than zero. If opponent uses mixed strat, ~~it would be~~ and is a minimaxer, it would be zero. BUT: if there is a lag between choice of strategy (by random device) and execution, during which it might be "found out" and long enough for counter to be prepared, then this minimum gain will be greater than zero! (this assumes that game is normalized in sense that strats are announced and executed together--but not necessarily chosen at same time).

Note: vN and M could have used notion of mixed strat in connection with minorant and majorant games, getting different results; but they didn't bother to.

A measure of "probable" gains from intelligence--not merely minimum--could be achieved by ruling out "unlikely" (e.g., dominated) enemy strategies, discounting "less likely" ones in advance.

THUS: DON'T MINIMAX REGRET, BUT COMPARE BEST OUTCOME FOR EACH COLUMN WITH THAT OF STRATEGY THAT YOU WOULD USE IN ABSENCE OF AN ESTIMATE OF INTENTIONS: MINIMAX, MAXIMAX, ETC. MINIMUM DIFFERENCE GIVES MINIMUM GAINS FROM PERFECT INTELLIGENCE. IF MAXIMUM DIFFERENCE IS SMALL, INTELLIGENCE ISN'T WORTH MUCH. i.e. if preferred that is [illegible] to

31. The opponent's perception of the payoff matrix and his consequent intentions may be much less ambiguous than our own estimate of the matrix--i.e., our guess as to the proper counter to the strategy we expect him to use. vN's division of the uncertainties may not correspond to the actual split.

An estimate of intentions requires us to know opponent's view of the payoff matrix (as well as knowing it ourselves). BUT: the possibility that opponent may misperceive payoff matrix increases the possibility that intelligence will be valuable--i.e., that it will pay to find him out--since it increases the probability that he will be found playing (what we regard as) a lousy strategy (assuming we continue to trust our own estimate). (In non-zero-sum game, there is a chance to change his strategy in our--mutual--favor by "setting him stragith" on payoff matrix; in both zero- and non-zero-sum games it could pay to delude him.)

32. Actual doctrine on counterespionage (CHECK) is to create ambiguity by conflicting indications, not by denying all indications to the enemy (should also have a theory as to what he is likely to do when confronted by this ambiguity).

32. The <u>reason</u> vN and M "divide the difficulties" as they do --ambiguity as to opponent's strategy, unambiguous payoff elements--is invalid; it is that they ignore ambiguity that does not arise from game considerations; they assume "nature" behaves probabilistically. If we reject this <u>belief</u> of theirs (they do not present it merely as a convenient assumption), we may conclude that it is not always a convenient assumption.

33. LIMITED WAR PROBLEM. Certain strategies have relatively unambiguous (not certain) results. e.g. launching preventive attack; it is virtually certain to evoke retaliation; uncertainties as to number of bombers and missiles we get through, accuracy of bombs, their speed of retaliation, effectiveness of our defenses (which have warning), accuracy of their missiles...may be represented by probabilities.

iA

But other strategies have ambiguous results: e.g. detterrence of various forms, or disarmament; their uncertainties, depending on what Soviets do, are much harder to calculate; the favorable outcomes are much better than the mathematical expectation under the preventive attack (unlike 1948, say) (and <u>for us</u> their worst outcomes aren't much worse; situation is different for Russians, which illustrates usefulness of this analysis), but it is hard to say whether they are more or less likely than other outcomes.

34. Cigaret ads; filtertips, nicotine and cancer. People buy filter-tips on the basis of expectations that they will never be able to check; hence there is no problem to influencing their behavior by creating false expectations. Compare with other expectations that can't be checked, at least until too late: consequences in afterlife; effect on internal organs; influence of internal organs (Carter's Little Liver Pills in both cases); "we could have won last time if..."

Note: expectations in the form of probability distributions may be very resistant to change, especially the more rectangular the distribution. Thus, expectation of a mixed strategy can persist in a repetitive game (Brown; Cournot) where expectation of a pure strategy wouldn't.

35. Possible influences on ambiguity: a) the shape of an "objective" probability dist; between two urns with known distributions, S might act as though the one with the more rectangular distribution were more ambiguous. b) repetition of uniqueness of the choice; S might act as though the "next ball" were ambiguous, but the distribution importance of the next 1000 balls were not ambiguous (in experiment, might act differently if this we e the "only chance to win" --or "the only chance to lose"--than if he knew he would have many opportunities (not necessarily of the same sort). In each case, check the amounts he would pay for and against an event.

36. Experimental framework. Let A_x be the amount a subject will pay for the gamble: aAO. (a is an amount constant throughout ; say \$10). A_y is the amount he will pay for OAa; i.e., the maximum amount he will bet "against" A (or on A'). For given a, there will be associated with every event A a vector (A_x, A_y).

The Savage axioms imply:

for no A,B: $(A_x, A_y) < (B_x, B_y)$

Crucial experiment: find A (ambiguous) and B (unambiguous) for which $A_x < B_x$ and $A_y < B_y$. * (or vice versa, for wishful thinker*

37. Hypothesis: that perferences among gambles depend on variance as well as mathematical expectation. But this violates Sure-Thing Principle. or admissibility

a) Suppose preference function linear in m.e. and variance, and positive weight given to variance (risk-taker):

I	1	1
II	.99..	1

II > I, which violates admissibility

b) Negative weight to variance (conservative).

I	1	1
II	1	1.000..1

I < II, which violates admiss.

c) Suppose lexicographic ordering: if m.e.'s are the same, negative eight to variance:

	½	½		½	½
I	10	20	III	10	20
II	20	10	IV	20	20

I = II, but III < IV, which violates Milnor's axiom (Rubin? Savage?)

compare with Tintner

$Y > 30$ $\quad$ $Y > 15$

$B < 45$

iff B is "not chosen"

$\Rightarrow$ $R + Y > B$

$\frac{Y+B}{2} = R$ given

$\frac{R + 2Y}{2} > R$

$2Y > R$

$Y > \frac{R}{2}$

$\frac{R}{2} + B < 2R$

$B < \frac{3R}{2}$

45 45

Y = 1

	Pess	Wishful	Neutral
If S has won	I i II III i IV	I i II III i IV	✓ ✓
If S didn't win	II p I IV p III	II p I IV p III	✓ ✓
Don't know	I p II IV p III	II p I III p IV	

	S won	Didn't win	Don't know
Pess	I i II III i IV	II p I IV p III	I p II IV p III
Wishful	I i II III i IV	II p I IV p III	II p I III p IV

R 30	Y 0–60	B 0–60	Min	Max	
a	0	0	$\frac{1}{5}a$	a	✓
0	a	0	0	$\frac{2}{3}a$	
a	0	a	$\frac{1}{3}a$	a	✓
0	a	a	0	$\frac{4}{5}a$	

$B = 0$

Min	Max	
$\frac{1}{3}a$	a	✓
0	$\frac{2}{3}a$	
$\frac{1}{3}a$	a	✓
0	$\frac{2}{3}a$	

$B = 60$

Min	Max
$\frac{1}{5}a$	$\frac{1}{3}a$
0	$\frac{2}{5}a$
$\frac{3}{5}a$	a
$\frac{4}{5}a$	$\frac{4}{5}a$

$B = 0,\ Y = 60$

Min	Max
$\frac{2}{3}a$	
$\frac{2}{3}a$ ✓	
$\frac{1}{3}a$	
$\frac{2}{3}a$ ✓	

	R	K	S
I	a	b	b
II	b	a	b
III	a	b	a
IV	b	a	a

If S has won: $\frac{\mathrm{I} : \mathrm{II}}{\mathrm{III} : \mathrm{IV}}$

If S didn't win: $\frac{\mathrm{II}\ p\ \mathrm{I}}{\mathrm{IV}\ p\ \mathrm{III}}$
(in fact, if S did not "win")

BUT: if I don't know:

conservation $\frac{\mathrm{I}\ p\ \mathrm{II}}{\mathrm{IV}\ p\ \mathrm{III}}$

R+D and ambiguity

see R-333

How many prototypes should be developed to early test stage? This may depend in the degree of ambiguity ~~xx~~ about the results. (Possible that with subjective probs well-defined, you would never test more than one at ~~xx~~ a time). (You <u>must</u> consult yourself on degree of ambiguity)

Again: does rationale for trying "different" solutions ("vulnerable to different uncertainties") really depend on assumption of ambiguity?

R&D as: a) choosing under ambiguity Klein urges them to <u>recognize</u> amb.
b) reducing ambiguity. Klein urges them to reduce amb.

Two types of wastes: a) acting as though prob were definite, when in fact they are ambiguous. (b) accepting ambiguity, and acting taking it into account, when it is possible <u>reduce</u> it.

Tests vs calculations: reduce ambiguity?

It is not enough to rely on your own feelings of ambiguity, subjective uncertainty, to know when to produce alternative prototypes. Consult record of R&D to know when expectations are "likely" to be contradicted by experience. (Like defining "objective ambiguity" by amount of controversy, difference in predictions; note that Savage designed minimax regret case for just that situation in which there was irreconcilable difference of opinion among respectable consultants--i.e., "objective ambiguity"--almost certainly "subjective ambiguity" so far as the decision-maker were concerned.) (But my people don't minimax regret.) ~~do they?)~~

Whether or not to pursue two or more parallel lines will depend on immediate cost of doing so, of possible "surprises" (ambiguity) and the importance of the surprises. ((MAY BE QUITE SOUND TO IDENTIFY--as Savage does--SURPRISE WITH THE PRE*EXISTENCE OF AMBIGUITY; this suggests that you should not look at focus outcomes in region where potential surprise is 0, but only when p.s. is positive. Where potential surprise is everywhere 0, maximize expected utility.))

Klein proposes easy rule: do it whenever it is cheap. But harder problem: how long should you keep doing it? Should you do it if it is expensive even in the beginning? Should you stop doing it as soon as it becomes expensive? (This may call for a concept of "degree of ambiguity"; or "potential surprise"?)

Some confusion arising from following fact: a) R&D decisions are designed to reduce uncertainty-risk-ambiguity. (b) R&D decisions are taken, at each stage, under uncertainty-risk-ambiguity. Two theories called for: a) the costs of ambiguity, and how to reduce it; b) how to act given a certain degree of ambiguity.

LEVELS OF AMBIGUITY

15 R, 15 B $\quad$ Δ: 5 R, 5 B $\quad$ S: 1 R, 1 B $\quad$ a) How much to bet, to win $1

U_{I} $\quad$ U_{II} $\quad$ U_{III} $\quad$ b) " " " " " " $100

Assuming Hurwicz behavior over ambiguous sets of events: the parameter α could be a monotonic function of "level of ambiguity."
~~increasing~~ if α applies to mihimum, $1-\alpha$ to maximum, then α could be increasing function of ambiguity for ~~wishful person, decreasing function for~~ pessimist, decreasing for wishful person.

In theory of statistical inference: is it assumed that the occurrence or non-occurrence of event A gives information only about ~~xh~~ the relative likelihoods of A vs. -A?

Or is it possible that it gives information about the relative likelihoods of events that make up -A?

(It appears that in the case of ambiguous events, the latter is true. Knowing that B did not occur, --on the basis of a random draw--gives information about the relative likelihood of R vs. Y)

The Zeuthen hypothesis (large differences in critical risk) can simply not be deduced from his assumptions. If it seems plausible, it must be because he (and I) supplies an additional, implicit assumption: perhaps, an assumption on a "typical" degree of uncertainty about opponent's payoffs. or expectations.

My ambiguity hypothesis:

If some events are ambiguous, ~~that~~ then each action has associated with it a range of outcomes, within which probabilities are not assigned. On the other hand, if some events are not ambiguous (say, with respect to each other and to the group of ambituous events), their utilities enter into the range of outcomes weighted by their subjective probabilities. These "constants" will be added to each possible outcome within the range of ambiguity; e.g., they will be added to the minimum and to the maximum. ~~xxxxxxxxxxxxxxxxxxx~~ The decision would be the same no matter whether: a) subject applies Hurwicz criterion to the ambiguous outcomes, then applies subjective probabilities; or (b) applies s.p. where applicable, then computes max and min for whole action and applies Hurwicz criterion. ((But suppose there are two groups of ambiguous outcomes, with different sp. applied?))

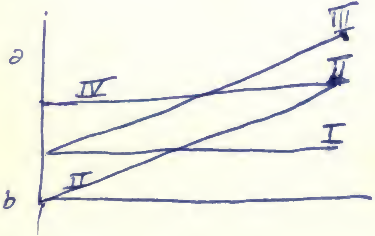
a	b	b
b	a	b
a	b	a
b	a	a

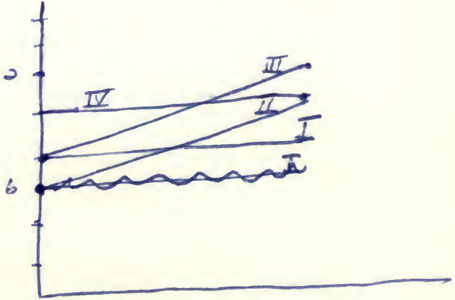
$$\frac{a}{3} + \frac{2}{3}(b \sim b) \qquad \frac{a}{3} + \frac{2}{3}b$$

$$\frac{b}{3} + \frac{2}{3}(b \sim a)$$

$$\frac{a}{3} + \frac{2}{3}(b \sim a)$$

$$\frac{b}{3} + \frac{2}{3}(a \sim a)$$





$\overset{30}{R} \qquad \overbrace{Y \quad B}^{60}$

Savage: Knowing that Black is "impossible" (will not come up--as a result of random factors; or did not come up in a random draw) may give knowledge about the relative likelihood of Red and Y ellow. i.e., if information is given that indicates that Black is "less likely" than Red and Yellow together,

given: $\frac{Y+B}{2} = R$

Suppose: $R + Y > B$ (B "not chosen" in a random draw).

Then, $\frac{R+2Y}{2} > R \Rightarrow 2Y > R \Rightarrow Y > \frac{R}{2}$ or $Y > \frac{15}{90}$

$\frac{R}{2} + B < 2R \Rightarrow B < \frac{3}{2}R \Rightarrow B < \frac{45}{90}$

(The pount here seems to be that the mere information that "black did not come up" tells more about the relative "likelihood" of Yellow than it does about Red, whose relative likelihood is in some sense "known", and unaffected by the information about the draw.)

① a) $a\alpha b > a\beta b \Longrightarrow a\bar{\beta} b > a\bar{\alpha} b$ | $\alpha > \beta \Rightarrow \bar{\beta} > \bar{\alpha}$

b) $a\alpha b = a\bar{\alpha} b$, $a\beta b = a\bar{\beta} b \Longrightarrow a\alpha b = a\beta b$ | $\alpha = \bar{\alpha}$, $\beta = \bar{\beta} \Rightarrow \alpha = \beta$

c) $a\alpha_1 b > a\beta_1 b$, $a\alpha_2 b > a\beta_2 b \Longrightarrow a(\alpha_1 \cup \alpha_2) b > a(\beta_1 \cup \beta_2) b$

Ex. 1 $\quad aR_1 b = aB_1 b$, $aR_2 b = aB_2 b$; $aR_1 b \overset{?}{=} aR_2 b$ (b)

Ex. 2 $\quad \cancel{aR_1 b = aB_1 b}$, $aR_2 b = aB_2 b$, $aR_1 b < aR_2 b$; $aB_1 b \overset{?}{>} aB_2 b$ (c)

Ex. 3

	$\overbrace{\text{Red}}^{100}$	Black	50 Yellow	50 Blue
I	a	a	b	b
II	b	b	a	a
III	a	b	b	b
IV	b	a	b	b
V	b	b	a	b
VI	b	b	b	a

I = II, III = IV, V = VI;

III $\overset{?}{=}$ IV $\overset{?}{=}$ V $\overset{?}{=}$ VI

Proof:

$$\begin{matrix} b & a & b & b \\ b & b & a & b \end{matrix} \longrightarrow \begin{matrix} b & a & b & a \\ b & b & a & a \end{matrix} \longrightarrow \begin{matrix} b & a & b & a \\ a & a & b & b \end{matrix}$$

$$\longrightarrow \begin{matrix} b & b & b & a \\ a & b & b & b \end{matrix} \longrightarrow \begin{matrix} b & b & a & b \\ b & a & b & b \end{matrix}$$

EX. 4

	30 (31) Red	Yellow	Black
I	a	b	b
II	b	a	b
III	a	b	a
IV	b	a	a

(Yellow + Black: 60)

$I \geq II \; ; \; III \geq IV$

30	60
a	b
b	b
a	b
b	a

EX. 5 $\;\; a \quad b \quad a \qquad U_I: 5R, 4B \; (100) \qquad U_{II}: 49R, 51B$

$\$1 \cdot R_1 \; \$0 > \$1 \cdot R_2 \cdot \$0 \; ; \; \$40 \cdot R_1 \cdot -\$30 \overset{?}{>} \$40 \cdot R_1 \cdot -\$30$

EX. 6 $\; aR_1b = aB_1b = c \;\; aR_2b = aB_2b = d \; : \quad c \overset{?}{=} d$

Arrow hyp.

$\widetilde{1,0},\ 2$ $\qquad$ $\widetilde{1,2},\ 0$

R,	Y,	B
a	b	b
b	a	b
a	b	a
b	a	a

1,0,2; R	1,2,0; R	1,0,2; Y	2,0: Y	0,2: B	0,2; B
a	a	b	b	b	b
b	b	a	a	b	b
a	a	b	b	a	a
b	b	a	a	a	a

"Given B" should have I & II, III & IV

"Given $\bar{B}$" (as result of random draws) : $p(Y) = \frac{2}{3}$, $p(R) = \frac{1}{3}$; $p(B) = 0$

I assume $R + Y \geq B$; $Y + B = 2R$ given;

$\Rightarrow Y \geq \frac{R}{2} = 1/6$, $B \leq \frac{3R}{2} = \frac{1}{2}$. (Y or B can be only 2/3 or 0).

Suppose ~~a~~ priori $p(B) = f\left(\frac{p_R}{p_Y}\right)$

So that if B is "unlikely" (wasn't drawn) then ~~B~~ R is "probably" > Y.

($p(B)$ is correlated with $\frac{p_R}{p_Y}$; same effects cause Y + B).

But if this can't be translated into a priori <u>probs</u> on R, Y, B ....

(i.e. person could <u>have</u> this sort of <u>conditional</u> prob assumption — even precisely? — and still choose to violate Sure-Thing.)

Arrow: Assume both know that opponent knows one's payoffs & expectations. This leads to equilibrium pt. solution — which may require mixed strategies.

(?)

-1, 1	1, -1	
1, -1	-1, 1	$\frac{1}{2}$
	$\frac{1}{2}$	

1, 1	1, 10
10, 1	0, 0

1, 1	-10, 0
0, -10	-1, -1

Talk with Schelling, Aug. 22, 1961

1. Consider bets on the proportion of Red to Black in the urn. Schelling assumes that subjects would behave as though subject to Fisher's Uncertainty of Rank B: they would assign probabilities (equal) to the different proportions. But my main proposition is that THEY WOULD TREAT THESE PROPORTIONS AS AMBIGUOUS, WOULD NOT ASSIGN PROBABILITIES TO THEM, although some might be less reasonable than others (if samples had been drawn) and some would be excluded.

2. If more than one "best guess," with ~~probxxsayx~~
then the "best guess" distribution can be regarded as the weighted composite distribution corresponding to the mix of these distributions. If the weights added to unity, we would have an overall composite prob distribution. But if they didn't, min and max would apply.

((Another possibility: classes of distributions, ambiguous within class but with weights assigned to each class.))

3. ((My <u>evidence</u> is adequate only for inferences about <u>normative</u> rules; though I would hope it had descriptive significance for reflective behavior.))

4. ((What I expect to last is: (a) rejection of Savage axioms in situations of ambiguity; (b) influence of "Bad" and "good" possibilities in these situations. Less sure: particular formula.)) ((Question is: do people ask questions about their preferences for payoffs and, separately, about the likelihood of events, the likelihood of these likelihoods, etc...or do they

~~also~~ ask these questions and <u>also</u> ask what are "good" or "bad" <u>distributions</u>, possibilities? I.e., is their weighting of events in their calculations determined independently of their weighting of payoffs?

Should spell out how formula operates to give weighting of payoffs. Simple cases; complex case.

~~((NOTE: formula is as though~~

1. Fellner implies wts. are independent of payoffs, rely only on conf.
2. Does symmetric bet reveal probs (Ramsey)? Not necessarily.
Shows symmetric weights, not probs.
3. Exploit Shackle quote; but on "Who will win?" 6-1 — 1-6

Why the advice to "believe the worst," or weight the unfavorable information in conditions of low information? Why would this have greater survival value, why believe that choosing this way would have results "good enough" to survive or better than some other behavior, unless Nature were really malevolent?

In asking the question, we are implicitly assuming that the available information, though scanty, is produced "impartially," i.e., independently or randomly distributed with respect to our desires, our payoffs, our set of available actions. But in many situations, we should assume that the information we receive, (e.g., from subordinates, agents, friendly sources) is already biased in a direction corresponding to our wants; or, the way we perceive or evaluate it is. So that advice would merely tend to counteract this already existing bias in our perceived information. (It is not that Nature is against us, but that our

What can statistician say, in ambiguous situation, when he doesn't know the payoffs (so doesn't know "good" from "bad" distribution)?

He can say about which propositions he would make consistent bets; he might be able to say that he assigned 1/3 probability to Red; perhaps that he assigned .5 probability to Yellow having at least 1/16 probability.

I.e., he can make probability statements about the proportion of balls in the urn. But where he <u>isn't sure</u> about the probabilities (e.g., of probabilities) he should give us a <u>range</u> of possibilities about which he can make a definite statement (like, attach weight of 95% to proportion's being within this range, 5% to it's being outside it but within this range);

Then we, taking payoffs into account, can attach 95% weight to a Hurwicz maximum within this range, 5% to the Hurwicz maximum outside it, etc.

Typical way to behave; assign equal probabilities in case of ignorance as "best guess" distribution; then "adjust" these, for each act, by raising weight for "bad" outcomes, lowering it for "good," in such a way as to preserve sum of unity. ((Hypothesis: if ordering of outcomes is the same for each act, this should result in the same weights being applied to payoffs for each act.)) Note difference from Fellner, who would apply bias reflecting only confidence, not payoffs.

$\alpha = \frac{1}{4} \quad \rho = \frac{3}{4}$

6	0	0
0	6	0
6	0	6
0	6	6

(As though person started with "best guess" weights, then "biased them" in *direction* depending on payoff (+ conservatism) and *amount* depending on confidence in "best guess."

	Best guess dist $\rho = \frac{3}{4}$	Worst dist $(1-\rho)(1-\alpha)\ \frac{7}{16}$	Best dist $(1-\rho)\alpha\ \frac{1}{16}$	Weights attached to payoffs	Minimaxer: $\alpha = 0, \rho = 0$
I	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$			
II	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	$\frac{1}{3}, 0, \frac{2}{3}$	$\frac{1}{3}, \frac{2}{3}, 0$	$\frac{1}{4} \cdot 0 + \frac{2}{48}$ $\frac{1}{3}, \frac{7}{24}, \frac{9}{24}$	$\lambda \quad 0 \quad 1-\lambda$
III	$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$	$\frac{1}{3}, \frac{2}{3}, 0$	$\frac{1}{3}, 0, \frac{2}{3}$	$\frac{1}{3}, \frac{9}{24}, \frac{7}{24}$	$0 \quad 1 \quad 0$

HINDSIGHT AND FORESIGHT

Gerald Loeb: The present is always the hardest time to invest. In the past there were no uncertainties.

Why is ~~xxx~~ harder to make choices in the present to affect the future than to make choices in the present to "affect the past": i.e., hypothetical choices (chosen now) that would "obviously" have affected past and present beneficially if they had been taken in the past.

Obviously, we know more now; have better theories, have more data, new sources of information (diaries, other side's memoirs, etc.); perhaps broader data than available to any one past decision-maker. But that might merely complicate calculations, raise new uncertainties; why in general should it make choice easier, more obvious?

Why isn't choice "easier" and more "obvious" on the basis of less information, partial information?

We try to improve present decision-making by studying past decisions in the light of present information. Rather, like Roberta Wohlstetter, we should try to study past decisions in the light of past information, try to analyze problems of MAKING DECISIONS IN THE PRESENT. In particular, problems of NOISE AND UNCERTAINTY.

Why didn't we move early in Laos? Hyp (see Taylor): We don't move militarily unless alternative is to lose with certainty. Why? Perhaps: in early stage, when small commitment would win, other measures also look promising.

Evaluate this as a policy

Ambiguous strategy is handicapped by double-weighting its less favorable possibilities, versus "traditional" strategy with the same "best estimate" probability distribution and the same outcomes. WHY?

1. Suppose one suspects in the case of the ambiguous strategy that one doesn't really know its full range of outcomes, and outcomes could be much worse than one imagines. They could also be better; but if we assume sharply curved utility, we would want to give extra weight to bad possibilities. Hence, giving extra weight to the bad possibilities we can foresee is a way of taking some account of the worse possibilities we have failed to foresee.

2. Weighting bad possibilities would be a way of counteracting a wishful bias already suspected to be present in estimates of ambiguous outcomes. If ambiguous strategy is an innovation, its supporters may be especially wishful types; or at least, concerned to stress its good possibilities. Weighting bad possibilities might also be a guard against tendencies to wishfulness.

3. You can't explain this by adjusting the utility payoffs of the known outcomes. (a) Even if this worked, it would mean assigning these outcomes different utilities in ambiguous and in risky situations. (b) Utility numbers simply can't be assumed--consistent with assumed ordering--that ~~xxxx~~ will explain certain choices, for any probability numbers. (c) Wouldn't know how to adjust payoffs unless probability numbers are assumed; and they are in question in ambiguous situation.

Ambiguous strategy

4. Assumption that there are game elements; that opponent (Nature) is really malevolent, and that game is essentially zero-sum.

5. Always possibility that this form of behavior is suited to certain situations and is carried over into others by habit, without being really appropriate (e.g., suited to games but carried over into games against Nature).

Chipman and me

Two American missiles, both with observed frequencies of success =.5, with A having 100 trials, B having 10 trials. You are the President; which do you choose?

Two Russian missiles, same information, to go up first. President controls agent who picks Russian missile. Which does he choose?

(b) a wire is to be used either to heat something, or to act as a safety fuse.

((α MAY BE A FUNCTION OF EST$_x$, THE "BEST GUESS" EXPECTATION: WHEN THE "BEST GUESS" IS "GOOD" α MAY BE LOW OR 0: BUT WHEN THE BEST GUESS IS BAD, α MAY BE HIGH. WILL LOOK AS THOUGH PROBS OF AMBIGUOUS EVENT ARE BIASED TOWARD 50:50; but in comparison with a 50:50 unambiguous bet, α may still be less than $\frac{1}{2}$, so that unambiguous bet is preferred. Note that if $\alpha = \frac{1}{2}$, the "best guess" distribution of the ambiguous bet corresponds to the unambiguous distribution of the other bet, and the "best" and "worst" expectations average out to the "best guess" expectation, the subject will be indifferent between the two bets; he may be acting as though they offered the same distribution.

When can probabilities below a certain level be ignored?
When they ~~xxxxx~~ could make no difference to our decisions; this will depend on the range of possible payoffs (and, in general, the structure of payoffs). E.g., if the addition of 1% of the "maximum possible" payoff ~~xxxxx~~ to the expected value of some action could not affect its ordering--or if we are relatively indifferent between two actions which differ only by that amount--(relative to the effort of taking such possibilities into account) then we might as well ignore probabilities under 1%.

From the desk of . . .

FRAN IRIE

Dan Ellsberg:

TS documents 2114 and 2128 are now available to you in the TS office Room 2783-B. Please come pick them up as soon as possible at your convenience.

Thank you

Fran Irie,
Alternate TS Officer

measure α by taking Yent asymmetrical

(at least, show $\alpha < \frac{1}{2}$ or $\alpha > \frac{1}{2}$). 1

Regrets i $\left\{\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}\right\}$ Expected regret: $0-1$ $\quad 1 \quad\quad 1000$

$0-1 \quad 0$

$\alpha > 0$ for everybody

ambiguous

α

$$\begin{array}{cccc} & & 1 & 0 \\ i & \mathrm{I} & 1 & 0 \\ & \mathrm{II} & 0 & 1 \end{array}$$

$1 \quad 0 \qquad 1 \quad 5 \qquad 1 \quad 0 \qquad 1 \quad 2$

$0 \quad -5$

$$\begin{array}{ccc} \mathrm{I} & 1 & 1 \\ P \ \mathrm{II} & 0 & 2 \end{array}$$

— measured using coin

$(\alpha < \frac{1}{2})$

MESSAGE

DATE 22-9

TO Dan TIME 1:10 A.M. P.M.

DURING YOUR ABSENCE

~~MR.~~ Mrs. Baum

OF OL-22350 Ext. 7

TELEPHONE NUMBER

Diner's Club

RAND		OUTSIDE	
X	PHONED	X	PHONE ~~HIM~~ her.
	WILL PHONE AGAIN		WANTS TO SEE YOU
	CALLED PERSONALLY		RETURNED YOUR CALL

MESSAGE:

FORM 9 (REV. 2-53) BY ro.

10~~b~~	−1000	0	1001
1	1	−100	0
2	−98	99	99

regrets show what you would gain with various pieces of <u>info</u> compared to payoff to given action

0	−1001
−102	0
−99	[illegible]

1. "If my decision between I & II is the same if α is true as if α is false, then it should be the same if I am uncertain whether α is true or false."

This expresses strong dominance.

It ~~does not apply to~~ is not equivalent to Savage's Axiom.

	a	b	b
→	b	a	b

a	b	a
b	a	a

---

a — a — b

a — b — a

a — b

a — b

$\frac{1}{3}$

a	b	0
b	a	0
0	0	a
0	0	a

With coop., joint max with side-payments

To get a one-interpretation sol., sociol. and psychol. assumptions must be added.

How ~~in~~ information leads to non-coop game, while only requires knowledge of one's own matrix (cost of getting complete info too high).

Pure comp. doesn't imply complete info., but equal ignorance — Viner.

Firm maximizes some variable in a monotonic relationship to money.

See Dalkey, Economet. Jan. '51, p. 53

Solution — best that [illegible] can enforce, with complete information on payoffs. in non-coop, non-commun game.

4. Advertising is cut down in recession: built up windfully in boom, companies don't really know what it brings in, can't afford to be wasteful in slump. How explain cut in quantity & shift in content of ads in slump?

5 Now is period of uncertainty — bull or bear? Prices drift down because no one wants to buy — meanwhile, normal sales (not abnormal). [If drift produced definite expectation of down movement, then pressure to sell would develop, accelerate downswing.] What happens on upswing?

6. Different expectations for "low-priced" and "high-priced" stock.

7. $\alpha = \beta \Rightarrow \bar{\alpha} = \bar{\beta}$. $\alpha = 48R, (52B)$ $\beta = ?R(?B)$

$a\alpha b = a\beta b$, but $a\bar{\alpha}b > a\bar{\beta}b$.

8 $\alpha = 50R, (50B)$ $\beta = (?B)\ ?R$ $a\alpha b > a\beta b$ and $a\alpha b = c\beta b = c\bar{\beta}b$

$10R_1 o = 12R_2 o$ $c > a$ $\Rightarrow a\bar{\alpha}b < a\bar{\beta}b < c\bar{\beta}b = a\alpha b$ **cont**

$10B_1 o < 12B_2 o$ [but $a\bar{\alpha}b = a\alpha b$]

15. Production decisions may depend on some indicator other than one's own sales: e.g. stock market, employment, sales of consumer industry or retail outlets, ~~prices~~ outcome of elections, federal budgets; thus, expectations which influence decisions ~~are~~ may not be simple extrapolations of trend of ~~variables~~ one's own sales.

Some flucs are regarded as seasonal; expected, planned for, ignored (i.e. – do not stimulate new decisions); but unexpected amplitude or timing (perhaps due to coincidence of several "normal" flucs) may spur new decision; while an abnormal fluc may be ignored because it comes when a "normal" fluc is expected.

16. Bam doesn't have decision model.

17. Notion of SUBJECTIVE DOMINATION → role of expectations in non-zero-sum games Luce

18. General notion of ambiguous weights affected by payoffs — to which minimax or maximax is an approximation.

19. Instead of regarding economy as a computer, regard it as a set of problem-solving machines, differing in decision-criteria (under uncertainty), lags, info networks, etc.

20. Distinction of "Bayes principle"; info may not be "in form of prob. dist."; hence, <u>allow</u> <u>randomization</u> and <u>mixed strategies</u> in stat. decision problems and games.

21. Application to <u>info theory</u>.

22. Precise stochastic knowledge about the performance of a factor, with ignorance about its total <u>importance</u>, is like knowledge about <u>components</u> but ignorance of <u>systems</u>.

23. May still be possible and <u>useful</u> to measure utility with <u>unambiguous</u> events.

24. Uncertainty in ~~decision~~ group decision-making & welfare ec.

27. Degree of preference will influence: a) the ranking of party platforms by a voter (other than his preferred platform — platform in sense of a ranking of goals, not merely a set of simultaneous objectives).

→ b) the framing of a platform by party: when the relative undesirability of an uncongenial plank is weighed against the probable support it will bring (net) and the probable effect of this support on the election (i.e., whether or not it is "probably" needed).

Note: election represents a level-of-achievement type goal. Multi-step goal: promotion in school or bureaucratic hierarchy.

Increasing-step goals: yardage in a football game (10-yd intervals). military campaigns.

See Kaplan on (Military Worth) success functions. These amount to specific assumptions on preference maps: possibly useful if they correspond to important classes (not merely individuals); by recognizing differences, they take advantage of similarities.

On "median hurdle" (sound barrier) case: shifting classes

28. Concept of "reasonable doubt"; dependent on valuation of Type I and Type II errors. — not just a doubt "equal to" 1 in 1000, say.

29. Military estimate of "intentions" vs. "capabilities." Problem is to estimate intentions, but *allow* for their ambiguity. (*and* for ambiguity of *other* factors).

|| NOTE: in real game, ambiguity of payoff elements may be much *greater* than of opponents intentions.

|| NOTE: gains from *knowledge* of intentions may be much greater than gains from mixed "optimal" strat vs. pure "non-optimal" strat. This is *test* whether it is *worthwhile* to *reduce* specific ambiguities.

~~30. Why are do closed-end investment cos. sell at a discount from asset value?~~

Where the "value" is below the "level of aspiration" (e.g. "total annihilation but not defeat") there is greater pressure to estimate intentions — and influence them.

Can we (or "X" country) *afford* to be conservative (or wishful), given our level of aspiration?

Sure-thing

$\alpha > \beta$

$$(P)\quad \begin{array}{ccc} \alpha & \beta & \overset{\gamma}{\bar{\alpha} a \bar{\beta}} \\ a & b & c \\ b & a & c \end{array} \Rightarrow (P) \quad \begin{array}{ccc} & \beta & \\ \alpha & \beta & \gamma \\ d & e & f \\ e & d & f \end{array} \qquad \begin{array}{ccc} a & b & b \\ b & b & a \end{array}$$

If $\alpha = \frac{1}{2}$, then for any $\beta \neq \frac{1}{2}$, $\beta > \alpha > \bar{\beta}$ or $\bar{\beta} > \alpha > \beta$

which means if α is any event $= \frac{1}{2}$, then for any $\beta \neq \frac{1}{2}$,

either $(a \alpha b)\ p\ (a \beta b)$ or $(a \alpha b)\ p\ (b \beta a)$ i.e., must prefer to bet on α rather than to bet (at same odds) on β or else on $\bar{\beta}$.

e.g. α is red from urn with unknown ratio.

Must either prefer Red out of this urn to Red out of urn with known ratio (e.g. 49R-51B) or to Black.

And (hence) if R_U is preferred to R_K $p\%$ of the time, B_K must be preferred to R_U same $p\%$.

You are an Astronaut. You are in charge of picking, for each flight, whether an Atlas or a Titan will be used as booster. Each has had an observed reliability of .6; the Atlas has had 100 tests, the Titan 25. So with 95% confidence, the reliability of the Atlast is over .51, the reliability of the Titan is over .41. With 95% conf., the prob. that the Atlas will abort is...

(a) Your own flight will be second. Your worst enemy is scheduled to go first. If his missile doesn't abort, he will be first Man in Space. Which missile do you give him?

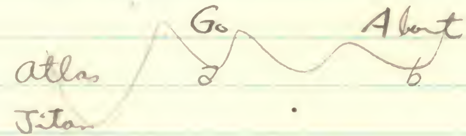
(b) Before you know results of his attempt, you must pick your own missile. Which do you pick?

(Conservative: pick Atlas for both. Can't be as confident that the Titan will abort; or that it will not abort. Yet this means you act "as if" you felt that Titan were "more likely" to abort, and also that it is "more likely" to succeed.)

Wishful: pick Titan for both.

Yet some will pick Titan for enemy, Atlas for self.

		Atlas	100	.6	.51
.6	.41	Titan	25	.6	.41
.6				.4	
				.4	



α - Atlas goes $\bar{\alpha}$ - Atlas aborts β Titan goes $\bar{\beta}$ - Titan abort

$a \alpha b$

$a \alpha b > a \beta b$ $\forall$ ride $\Rightarrow \alpha > \beta$

& $\bar{\alpha}\,\cancel{d}$

John Cohen & Mark Hansel: Risk and Gambling, the Study of Subjective Probability Longmans, Green & Co. London, 1956.)
(Prof + Lect of Psychology, Univ of Manchester).
articles by Jarrett-Goodnow in Am. Journ. of Psychol. + Journ. of Exp. Psychol., 1955-56
Ray Hyman, same, 1955-56
Preston & Baratta, "Auction Value of an Uncertain Outcome" AJP or JXP about 1946

Chipman: Stochastic Choice and Subjective Probability

Notion that an individiual in a <u>given</u> "state of mind" will be considered as having a strong ordering of the elements of the choice set, but that he has a prob dist over all possible statesof mind (i.e. permutations of the choice set). Assume finite set. Luce: prob of pref

Ramsey's notion of "ethoically neutral event"; if the preference of xAy over yAx is independent of the particular x and y, provided that one is absolutely preferred to the other. "This implies that the outcome of the event A is not itself an object of desire which would influence one's choice between x and y." p. 3 "Since our findamental notion is that of a prob dist over states of mind, we can say that if P(x,y) equals 1, then ~~P(x'x,y'x)~~ $P(x'\alpha y', y'\alpha x') = P(x\alpha y, y\alpha x)$, given $P(x', y') = 1$.

"The justification for the assumption of 'ethical neutrality' of beliefs is that one should be able to find a sufficiently interesting set E of events the outcomes of which are not themselves the objects of desire. It is plain, however, that if we are to include events such as 'Nixon will be the next president,' the assumption will no longer be warranted." 4

((But Chipman ignores possibility of reverse influence; events desirability being influenced by the payoffs--perhaps in some other gamble.))

$\qquad a \qquad\quad b$

Savage: if x is preferred absolutely to y. $(P(x,y)=1)$, $(x\alpha y)\ p\ (x\beta y)$ $\Rightarrow$ α is subjectively more probable than β.

In this case,

If $Q(\alpha,\beta)$ is the (objective) prob that α is considered more likely than β, then $Q(\alpha,\beta) = P(a,b)$. Likewise, for those states of mind in which y is preferred to x, preference of a over $b \rightarrow \beta$ is considered more probable than ~~β~~ α, so $Q(\beta,\alpha) = P(a,b)$. Since $P(x,y)$ is the prob of the first state of mind and $P(y,x)$ the second,

$$P(a,b) = P(x,y)\,Q(\alpha,\beta) + P(y,x)\,Q(\beta,\alpha)$$

which is Luce's decomposition axiom.

[Note: if $P(x,y)=1$, $P(y,x)=0$, then $Q(\alpha,\beta) = P(a,b)$. Since Q is supposed to be independent of the payoffs, it should be the same for all payoffs, depending <u>only</u> on α and β. But minimaxer: $(a\overset{x}{\alpha}b)\ p\ (a\overset{y}{\beta}b)$ for small a, b; but with ambiguous α, β and large a, b, $Q(\alpha,\beta) = P(x,y)$ may be closer to $\frac{1}{2}$; in the limit, $=\frac{1}{2}$, so that small increments (to b, say) could swing preference either way).

$$Q(\alpha,\beta) = \frac{P(x,y) + P(a,b) - 1}{2P(x,y) - 1}$$

Chapman: 3

Weak transitivity postulate:

For all $a, b, c \in A$, if $P(a,b) \geq \frac{1}{2}$, ~~then p~~ and $P(b,c) \geq \frac{1}{2}$, then $P(a,c) \geq \min[P(a,b), P(b,c)]$

(stronger than Marschak + Davidson's weak post. that $P(a,c) \geq \frac{1}{2}$)

Strong trans. post:

For all $a, b, c \in A$, if $P(a,b) \geq \frac{1}{2}$ and $P(b,c) \geq \frac{1}{2}$ then $P(a,c) \geq \max[P(a,b), P(b,c)]$

This is same as M + D's strong trans. post.

But suppose: α is drawing ball from with   Exp in knee.

Chipman: 4

Experiment: S chose between matchboxes, some of which contained a given proporition of heads and stems (60-40, 50-50, etc.), and some with unknown proporitions: but samples (7-3, 4-6, etc.) were drawn from the latter. Money bets. (three wagers: 25,-25; 0-25; 0, 25)

Results: 1. Between boxes with known dists, subjects always chose the better dist.

2. Ss violated the Strong Transitivity Post (Marschak and Davidson). i.e., the proporition of preference of a over c was less than the maximum of a over b or b over c (latter might be 1).

3. All subjects conformed to weak trans postulate.

4. Much violation of Luce's linearity axiom: that for all a,b in A, either P(a,c) greater or equal to P(b,c) for all c in A, or less than or equal.

5. Same individuals violated Luce's Independence of Irrelevant Alternatives axiom.

6. "One of the most striking features shown by the data is the tendency for individuals to bias unknown probabilities towards one-half." 22 I.e., where sample shows 7 heads, 3 tails, subject prefers box with <u>known</u> 70-30 dist; but where sample shows 3 heads, 7 tails, subject prefers that unknown box to one with known 30-70 dist.

Subjects strongly preferred a known 50-50 dist to an unknown dist with a 5-5 sample.

((The preference when math. expect is negative conforms to wishful

thinking; but behavior under high favorable probs doesn't. But maybe wishful thinking operates strongly (or only) when the problem is to avoid a loss. Or, perhaps these preferences depend on the scale of payoffs; perhaps with larger scale, subjects would have shown consistently wishful thinking.

Or, it may not show wishful thinking; choices suggest a Bayesian principle at work, assigning uniform probs to unknown events; except that they preferred 50-50 to 5-5 sample. Exp: Which ~~thexyxw~~ would they prefer, a 50-50 box or a box with completely unknown dist, no sample drawn? This would be a direct test of Bayesian hypothesis.

Kaysen suggests a preference for higher variance? Is there any basis for saying that the unknown box has a higher variance? I don't think so. Anyway, we could test directly by seeing if subject preferred a box with a higher <u>known</u> variance to a box with a lower known variance, or with unknown variance (I think we must say that sample box has <u>unknown</u> variance; all we have is a maximum likelihood estimate of the mathematical expectation). ((((()))))

(((((EUREKA! Doesn't Luce independence of probs and utilities imply:

$(a\alpha b)\, i\, (b\alpha a),\ (a\beta b)\, i\, (b\beta a) \Rightarrow (a\alpha b)\, i\, (a\beta b);$ and $(b\alpha a)\, i\, (b\beta a)$?

This experiment is counter-exampbe. With 5-5 sample, subjects would be indifferent betwøen betting on heads or stems; but they would prefer to draw from a box with a known 50-50 dist. In fact, they might prefer to draw from a 49-51 box, though this wasn't tested.)))

α – 50% known dist β – 50-50 sample

i.e. $(a\alpha b)\, i\, (b\alpha a),\ (a\beta b)\, i\, (b\beta a)$. BUT ~~a~~ $(a\alpha b)\, p\, (a\beta b)$

Chipman 6

??Did these preferences between known-unknown dists hold for negative, positive and ero math expectations? Or was there a difference? Would they be the same for different levels of payoff?

Chipman mentions that the data on this pair "suggest that a minmax principle is at work."

Chipman concludes that subject does not merely characterize event by a probability, but that some other parameter is involved; he concludes that the other parameter is sample size (i.e., size of sample drawn from unknown dist). ((My opinion: the other parameter is somethinglike confidence or ambiguity, with perhaps a third parameter of importance. Sample size is an indicator of confidence and ambiguity, but not always applicable, rarely well-defined).))

"Our basic approach has been that, whether the subject is conscious of it or not, he acts as if any event could be characterized by two numbers, one of them a frequency and the other a sample size." "For example the casual poker player will not bother to figure out, in a game of five-card stud, whether the objective probability (in the conventional sense) of playing for a straight is higher than that of playing for a full house; ...In the simplest case, we may say that he simply observes how many times he won or lost when he previously faced this alternative, and acts according to the observed frequency and sample sie." ((NOTE: C. fails to say that he

acts according to the frequencies he has previously observed to occur, but according to his previous wins and losses. This is probably true; subject is influenced by his wins and losses, though Chipman generally hints that he is influenced only by his observations))

In any case, "the scientist has no choice but to discover the subject's theory in order to predict his behavior."

((Counter-Luce: suppose minimax behavior consisted not in acting as though worst outcome were certain, but as though it was more probable than otherwise, depending on importance. A strict minimaxer, will be indifferent between all gambles offering the same outcomes, whatever the events; i.e., P(a,b) will always be 1/2, where a and b are gambles offering the same outcomes. But a quasi-minimaxer, above will approach indifference, as the payoffs get larger (Chipman's exp. suggests-- as they get negatively large, between gambles involving same payoffs and ambiguous events; thus

SUBJECT CAN BE SAID TO ACT AS IF HE ATTACHED PROBABILITIES TO PAYOFFS, NOT EVENTS (in the limit); i.e., payoff gets a weight (which need not equal 1--Hurwicz) which depends only on its relative size.